QCD tests for Quasilocal Quark Model*

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Abstract

We perform the QCD testing of the Quasilocal Quark Model (QQM) based on Operator Product Expansion (OPE). The quark current correlators calculated in framework of the model, are compared to their OPE in QCD at intermediate energies. The QQM provides a reasonable resolution for mass spectrum of parity doublers in scalar and vector meson channels.

1. Introduction

Effective quark models are widely used to simulate main features of nonperturbative QCD at low and intermediate energies while having advantages to be more tractable in calculations. The quality of a simulation has to be controlled by a number of QCD tests. First of all, an effective model should reproduce the symmetries of QCD. Second, the chiral and conformal symmetries are broken in QCD eventually leading to formation of quark and gluon condensates respectively. The latter one should be also embedded into a model. Third, quark current correlators calculated in framework of a model, are to be matched to the Operator Product Expansion (OPE) [1] at intermediate energies. One could mention also the heavy mass matching for $m_q \gg \Lambda_{QCD}$ and the reproduction of chiral and scale anomalies which however are not involved in our discussion.

The requirements enumerated above are quite severe. For example, the Chiral Perturbation Theory containing the pseudoscalar degrees of freedom only does not pass all the QCD tests: matching to the OPE gives wrong results for this theory. In the present report we perform the QCD testing of the so called Quasilocal Quark Model (QQM), which admits as linear realization [2, 3, 6] as non-linear one [8, 9, 10]. It fulfils the matching to the OPE rather successfully.

In this talk we deal with the linear realization of QQM which represents an extension of NJL-type models [11, 12, 14, 15, 18, 19] and allows to describe not only ground states of scalar (S), pseudoscalar (P), vector (V), and axial-vector (A) mesons but also their radial excitations known from Particle Data [20].

The minimal structure of SP, $SU(2)$ QQM was discussed in [2, 6] and the VA, $SU(2)$ case was outlined in [21]. Here we consider the $SPVA$, $U(3)$ QQM [22]. In the Euclidean space the relevant Lagrangian has the form:

$$L = i\bar{q}\left(\hat{\partial}+\hat{m}\right)q + \frac{1}{4N_f N_c\Lambda^2}\sum_{k,l=1}^{2}\mathrm{Tr}\left\{a^a_{kl}\sum_{j=1}^{2}\bar{q}f_k(\tau)\Gamma^a_j q\bar{q}f_l(\tau)\Gamma^a_j q \right.$$
$$\left. +b^a_{kl}\sum_{j=3}^{4}\bar{q}f_k(\tau)\Gamma^a_j q\bar{q}f_l(\tau)\Gamma^a_j q\right\}. \quad (1)$$

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a^a_{kl} and b^a_{kl} represent here symmetric matrices of real coupling constants in SP and VA case respectively. Symbols Γ^a_j mean:

$$\Gamma^a_1 \equiv \lambda^a, \quad \Gamma^a_2 \equiv i\gamma_5\lambda^a, \quad \Gamma^a_3 \equiv i\gamma_\mu\lambda^a, \quad \Gamma^a_4 \equiv i\gamma_5\gamma_\mu\lambda^a; \qquad a = 0, ..., 8\,, \tag{2}$$

where

$$\lambda^a = \frac{1}{\sqrt{2}}\lambda^a_{G-M}\,, \quad a = 1, ..., 7\,, \tag{3}$$

$$\lambda^0 = \frac{\lambda^0_{G-M} + \lambda^8_{G-M}}{\sqrt{6}}\,, \quad \lambda^8 = \frac{-\lambda^0_{G-M} + \sqrt{2}\,\lambda^8_{G-M}}{\sqrt{6}}\,, \tag{4}$$

with λ^a_{G-M} being the standard set of Gell-Mann matrices. The current quark mass matrix is $\hat{m} = diag(m_u, m_d, m_s)$. In the sequel we adopt the exact isospin symmetry $m_u = m_d$. The symbol $\tilde{u}$ will stand everywhere for the u, d, $\bar{u}$, $\bar{d}$ quarks. The symbol $\tilde{s}$ will denote s or $\bar{s}$ quarks. We choose the polynomial form-factors to be orthogonal on the unit interval:

$$\int_0^1 f_k(\tau) f_l(\tau) d\tau = \delta_{kl}\,, \tag{5}$$

$$f_1(\tau) = 2 - 3\tau; \qquad f_2(\tau) = -\sqrt{3}\,\tau; \qquad \tau \equiv -\frac{\partial^2}{\Lambda^2}\,. \tag{6}$$

The parameter Λ is a four-momentum cutoff for virtual quark momenta in quark loops. N_c denotes a number of colors and $N_f = 3$ is the number of quark flavors.

Let us comment the approximations which will be used to derive the meson characteristics: namely, the large N_c and leading-log ($\ln\frac{\Lambda^2}{\mu^2} \gg 1$) approximations. The first one is equivalent [23, 24] to neglecting of meson loops. The second one fits well the quarks confinement as quark-antiquark threshold contributions are suppressed in two-point functions in the leading-log approximation. The accuracy of this approximation is controlled by the magnitudes of higher dimensional operators neglected in QQM, i.e. by contributions of heavy mass resonances not included into QQM. All these approximations are mutually consistent.

We will work with bosonized action. Thus, one introduces auxiliary $SPVA$-fields following the standard procedure:

$$L_{aux} = i\bar{q}\left(\hat{\partial} + \hat{m} + \sum_{k=1}^{2} \varphi^a_{k,j}\Gamma^a_j f_k(\tau)\right) q + N_c N_f \Lambda^2 \sum_{k,l=1}^{2} Tr\,\varphi^a_{k,j}\,(c^a_{kl})^{-1}\,\varphi^a_{l,j}\,, \tag{7}$$

where $\varphi^a \equiv \sigma^a, \pi^a, \rho^a, A^a$ represents auxiliary S, P, V, A fields and c^a_{kl} denotes a^a_{kl} for SP-case and b^a_{kl} for VA-case.

For the cancellation of quadratic divergences of order Λ^2 the following parameterization of coupling constants is accepted in the SP-case:

$$8\pi^2\,(a^a_{kl})^{-1} = \delta_{kl} - \frac{\Delta^a_{kl}}{\Lambda^2}; \qquad \Delta_{kl} \ll \Lambda^2\,, \tag{8}$$

where the physical mass parameters Δ^a_{kl} satisfy the relations:

$$\Delta^m_{kl} = \Delta^0_{kl}\,, \quad \Delta^n_{kl} = \frac{1}{2}\left(\Delta^0_{kl} + \Delta^8_{kl}\right)\,; \qquad m = 1, 2, 3\,, \quad n = 4, 5, 6, 7\,. \tag{9}$$

The same conditions are valid for VA-case with the replacement

$$(a^a_{kl})^{-1} \to 2\,(b^a_{kl})^{-1}\,, \qquad \Delta^a_{kl} \to \frac{4}{3}\bar{\Delta}^a_{kl}. \tag{10}$$

The self-consistency of the mass spectrum turns out to impose the following scale conditions:

$$\Delta^i_{kl} \sim \ln\frac{\Lambda^2}{M_0^2}\,; \quad \bar{\Delta}^i_{kl} \sim \Lambda^2\,; \quad m^i_q\Lambda^2 \sim 1\,, \tag{11}$$

where M_0 is a dynamic quark mass.

2. Mass formulas from QQM with account of OPE

The expressions for the mass spectrum are displayed in [22]. Here we present the mass relations, which are independent of model parameters in the large-log approximation. Some comments are in order. The gluon anomaly is omitted in the present report. Thus, the η'-meson is not considered. However, we do not see any reason for appearance of $U_A(1)$ problem for excited states. Below on the prime denotes excited states and the symbol $(\eta)'$ means the first radial excitation of η-meson.

First of all, the Gell-Mann-Okubo relation holds in the model and the singlet state has no admixture of s-quark in $U(3)$ case:

$$m^2_{\alpha,\tilde{u}\tilde{u}} + m^2_{\alpha,\tilde{s}\tilde{s}} = 2m^2_{\alpha,\tilde{s}\tilde{u}}\,; \qquad m_{\alpha,\tilde{u}\tilde{u}} = m_{\alpha,singlet}\,. \tag{12}$$

Here $\alpha \equiv S, V, A; S', V', A', P'$. For P-case one has the $SU(3)$ relation:

$$m^2_{P,\tilde{u}\tilde{u}} + 3m^2_{P,\tilde{s}\tilde{s}} = 4m^2_{P,\tilde{s}\tilde{u}}\,. \tag{13}$$

All other relations presented below are derived within the framework of $SPVA$, $U(3)$ QQM.

For the ground SP-meson states one has:

$$m^2_{\sigma,\tilde{u}\tilde{u}} - 3m^2_\pi = m^2_{\sigma,\tilde{s}\tilde{u}} - 3m^2_K = m^2_{\sigma,\tilde{s}\tilde{s}} - 3\left(2m^2_K - m^2_\pi\right) \simeq 4\mathcal{M}_0^2\,. \tag{14}$$

Here and below the dynamic mass $\mathcal{M}_0 \equiv M_0\mid_{m_q=0}$. In the VA-meson sector the relations for ground states look as follows:

$$m^2_{a_1,\tilde{u}\tilde{u}} - m^2_\rho \simeq \frac{3}{2}\left(m^2_{\sigma,\tilde{u}\tilde{u}} - m^2_\pi\right)\,, \quad m^2_{a_1,\tilde{s}\tilde{u}} - m^2_{K^*} \simeq \frac{3}{2}\left(m^2_{\sigma,\tilde{s}\tilde{u}} - m^2_K\right)\,, \tag{15}$$

$$m^2_{a_1,\tilde{s}\tilde{s}} - m^2_\varphi \simeq \frac{3}{2}\left[m^2_{\sigma,\tilde{s}\tilde{s}} - \left(2m^2_K - m^2_\pi\right)\right]\,. \tag{16}$$

For the excited VA-meson states one has:

$$m^2_{a_1',\tilde{u}\tilde{u}} - m^2_{\rho'} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{u}\tilde{u}} - m^2_{\pi'}\right)\,, \quad m^2_{a_1',\tilde{s}\tilde{u}} - m^2_{K^{*\prime}} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{s}\tilde{u}} - m^2_{K'}\right)\,, \tag{17}$$

$$m^2_{a_1',\tilde{s}\tilde{s}} - m^2_{\varphi'} \simeq \frac{3}{2}\left(m^2_{\sigma',\tilde{s}\tilde{s}} - m^2_{(\eta)'}\right)\,. \tag{18}$$

As it was already mentioned, at intermediate energies the correlators of QQM can be matched [26] to the OPE of QCD correlators [1]. In the large-N_c approach the correlators of color-singlet quark currents are saturated by narrow meson resonances. In particular, the two-point correlators are given by the following sums:

$$\Pi^C(p^2) = \int d^4x e^{ipx} \langle T(\bar{q}\Gamma q(x)\bar{q}\Gamma q(0))\rangle_{planar} = \sum_n \frac{Z_n^C}{p^2 + m_{C,n}^2} + D_0^C + D_1^C p^2, \tag{19}$$

$$C \equiv S, P, V, A; \qquad \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5; \qquad D_0, D_1 = const.$$

The last two terms both in the scalar-pseudoscalar and in the vector-axial-vector channel D_0 and D_1 are contact terms required for the regularization of infinite sums. On the other hand the high-energy asymptotics is provided [1] by the perturbation theory and OPE. Therefrom the above correlators increase at large p^2,

$$\Pi^C(p^2) \mid_{p^2\to\infty} \sim p^2 \ln \frac{p^2}{\mu^2}. \tag{20}$$

Evidently the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotics.

Meantime the differences of correlators of opposite-parity currents rapidly decrease at large momenta [8, 26] (the chiral limit is considered below):

$$\left(\Pi^P(p^2) - \Pi^S(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right), \qquad \Delta_{SP} \simeq 24\pi\alpha_s\langle\bar{q}q\rangle^2, \tag{21}$$

and [1, 28]

$$\left(\Pi^V(p^2) - \Pi^A(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{VA}}{p^6} - \frac{m_0^2\Delta_{VA}}{p^8} + \mathcal{O}\left(\frac{1}{p^{10}}\right), \; \Delta_{VA} \simeq -16\pi\alpha_s\langle\bar{q}q\rangle^2, \tag{22}$$

where $m_0^2 = 0.8 \pm 0.2\,\mathrm{GeV}^2$ [29] and we have defined in the V, A channels

$$\Pi_{\mu\nu}^{V,A}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\mu p_\nu)\Pi^{V,A}(p^2)\,. \tag{23}$$

The vacuum dominance hypothesis [1] in the large-N_c limit is adopted.

Therefore the chiral symmetry is restored at high energies and the two above differences manifest genuine order parameters of CSB in QCD. As they decrease rapidly at large momenta one can perform the matching of QCD asymptotics by means of few lowest lying resonances that gives a number of constraints from Chiral Symmetry Restoration (CSR).

Expanding the meson correlators (19) in powers of p^2 one arrives to the CSR Sum Rules. In the scalar-pseudoscalar case (21) they read:

$$\sum_n Z_n^S - \sum_n Z_n^P = 0\,, \quad \sum_n Z_n^S m_{S,n}^2 - \sum_n Z_n^P m_{P,n}^2 = \Delta_{SP}\,, \tag{24}$$

and in the vector-axial-vector one (22) one obtains:

$$\sum_n Z_n^V - \sum_n Z_n^A = 4f_\pi^2\,, \quad \sum_n Z_n^V m_{V,n}^2 - \sum_n Z_n^A m_{A,n}^2 = 0\,,$$

$$\sum_n Z_n^V m_{V,n}^4 - \sum_n Z_n^A m_{A,n}^4 = \Delta_{VA}\,, \quad \sum_n Z_n^V m_{V,n}^6 - \sum_n Z_n^A m_{A,n}^6 = -m_0^2 \Delta_{VA}\,. \tag{25}$$

The first two relations are famous Weinberg Sum Rules, with f_π being the pion decay constant. The residues in resonance pole contributions in the vector and axial-vector correlators have the structure,

$$Z_n^{(V,A)} = 4 f_{(V,A),n}^2 m_{(V,A),n}^2\,, \tag{26}$$

with $f_{(V,A),n}$ being defined as corresponding decay constants.

In the SP case the residues in poles happen to be of different order of magnitude in logarithms,

$$\frac{Z_{\sigma,\pi}}{Z_{\sigma',\pi'}} = \mathcal{O}\left(\frac{1}{\ln\frac{\Lambda^2}{M_0^2}}\right), \tag{27}$$

and when the $\pi - a_1$ mixing is taken into account one derives:

$$Z_\pi \simeq \frac{m_{a_1}^2}{m_\rho^2} Z_\sigma \simeq 4\frac{\langle\bar{q}q\rangle^2}{f_\pi^2} \simeq -\frac{N_c \Lambda^4 \Delta_{22}(\sigma_1 - \sqrt{3}\,\sigma_2)^2}{12\pi^2 m_{\sigma'}^2 \sigma_1^2 \ln\frac{\Lambda^2}{M_0^2}} \cdot \frac{m_{a_1}^2}{m_\rho^2}\,;$$

$$Z_{\pi'} = Z_0 - Z_\pi \simeq Z_{\sigma'} = Z_0 - Z_\sigma\,, \qquad Z_0 \equiv \frac{N_c \Lambda^4}{2\pi^2}\,. \tag{28}$$

The second CSR Sum Rules constraint results in the estimation for splitting between the σ'- and π'-meson masses,

$$Z_0(m_{\sigma'}^2 - m_{\pi'}^2) \simeq 24\pi\alpha_s \langle\bar{q}q\rangle^2. \tag{29}$$

In the vector-axial-vector case all residues are found to be of the same order of magnitude in contrast to the scalar-pseudoscalar channel. They read:

$$\widetilde{Z}_\pi = 4 f_\pi^2 \simeq -\frac{N_c \Lambda^4 (m_{a_1}^2 - m_\rho^2)(6 m_\rho^2 \ln\frac{\Lambda^2}{M_0^2} + 3\bar{\Delta}_{11} + 2\sqrt{3}\bar{\Delta}_{12} + \bar{\Delta}_{22})}{32\pi^2 m_\rho^2 m_{a_1}^2 m_{a_1'}^2 \ln\frac{\Lambda^2}{M_0^2}}\,;$$

$$Z_\rho \simeq \frac{m_{a_1}^2}{m_{a_1}^2 - m_\rho^2} Z_\pi\,, \qquad Z_{a_1} \simeq \frac{m_\rho^2}{m_{a_1}^2 - m_\rho^2} Z_\pi\,;$$

$$Z_{\rho'} \simeq \frac{Z_1}{m_{\rho'}^2}\,, \qquad Z_{a_1'} \simeq \frac{Z_1}{m_{a_1'}^2}\,, \qquad Z_1 \equiv \frac{3 N_c \Lambda^4}{16\pi^2}\,. \tag{30}$$

The relation for $\widetilde{Z}_\pi$ is a constraint on effective coupling constants of the QQM $\bar{\Delta}_{kl}$. The first and the second Sum Rules are fulfilled identically. The third one takes the form:

$$Z_1(m_{a_1'}^2 - m_{\rho'}^2) \simeq 16\pi\alpha_s \langle\bar{q}q\rangle^2. \tag{31}$$

The fourth Sum Rule gives in the large-log approach:

$$m_{a_1'}^2 \simeq m_{\rho'}^2 \simeq \frac{m_0^2}{2}\,. \tag{32}$$

As it is seen from the Eq. (32) the last Sum Rule fails for QQM with the ground and first excited sets of VA mesons.

The relations (29) and (31) constrain the QQM parameters following from the OPE. Having as an input $\Lambda = 1000$ MeV and

$$\mathcal{M}_0 = 2\sigma_1 = 320\,\mathrm{MeV}\,, \quad \langle\bar{q}q\rangle \simeq -\frac{N_c\Lambda^2}{8\pi^2}(\sigma_1 - \sqrt{3}\,\sigma_2) = -(250\,\mathrm{MeV})^3, \tag{33}$$

one can fix

$$\sigma_1 = 160\,\mathrm{MeV}\,, \qquad \sigma_2 = -145\,\mathrm{MeV}\,. \tag{34}$$

From the QQM relation

$$m^2_{a'_1} - m^2_{\rho'} \simeq \frac{3}{2}(m^2_{\sigma'} - m^2_{\pi'}) \simeq 3\sigma_1^2 + 2\sqrt{3}\,\sigma_1\sigma_2 + 9\sigma_2^2 \tag{35}$$

and Eq. (29), (31) one obtains the mass splittings

$$m_{\sigma'} - m_{\pi'} \approx 45\,\mathrm{MeV}\,, \qquad m_{a'_1} - m_{\rho'} \approx 60\,\mathrm{MeV}\,, \tag{36}$$

which prove a fast restoration of chiral symmetry. One can also estimate the required $\alpha_s \approx 0.9$ at one loop. It seems to be bearly compartible with perturbative calculations in QCD. On the other hand, in the vector channel the two-loop calculations [29] diminish considerably the value of required strong coupling constant $\alpha_s \approx 0.5 \div 0.6$.

Note that from Eq. (35) and (29), (31) one may obtain two independent estimations of the quantity:

$$\frac{m^2_{a'_1} - m^2_{\rho'}}{m^2_{\sigma'} - m^2_{\pi'}} \approx \begin{cases} 1.5 & \text{from QQM,} \\ 1.8 & \text{from CSR,} \end{cases} \tag{37}$$

which do not depend on any model parameters. The discrepancy amounts to 15%, i.e. it is within the large-N_c approximation. This shows that saturation of two-point correlators by two resonances is quite robust.

Finally, in the Appendix we display the Table with our fits for $SPVA$ meson masses and compare them with the corresponding experimental values [20]. A rather big discrepancy in predicting masses of ground scalar states is a general problem in phenomenology. In the case of QQM this might signify that large-N_c corrections are large or the leading-log approximation does not work well, i.e. the details of confinement are of importance in this case. Note, however, that the Extended Chiral Quark Model [8, 9, 10] fits better the scalar sector.

We conclude that the Quasilocal Quark Model reflects phenomenology of low and intermediate meson physics and passes QCD tests with the reasonable precision. It must be noticed that in the conventional NJL model the precision was worse substantially. Namely, in the SP channel one can derive an NJL-model estimation

$$\frac{Z_\sigma m_\sigma^2}{24\pi\alpha_s\langle\bar{q}q\rangle^2} = \frac{22}{9}\,,$$

whereas this ratio should be equal 1 from the second CSR Sum Rule Eq. (24).

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Table 1: The $SPVA$, $U(3)$ QQM masses of mesons and their first excitations (in MeV) for $\mathcal{M}_0 = 320$ MeV. $\tilde{u} \equiv u, d$ quarks.

Particle, th.	Particle, exp.	Input	Pred.	Experiment	Dif., %	Case
$Singlet$						
$\tilde{u}\tilde{u}$	π	140		135-140		P
$\tilde{s}\tilde{u}$	K	500		494-498		
$\tilde{s}\tilde{s}$	η		570	547.30±0.12	4	
$(Singlet)'$	$\eta(1295)$		1300	1297.0±2.8	<1	
$\tilde{u}'\tilde{u}'$	$\pi(1300)$	1300		1300±100		P'
$\tilde{s}'\tilde{u}'$	$K(1460)$	1400		1400-1460		
$\tilde{s}'\tilde{s}'$	$\eta(1440)$		1490	1400-1470	1	
$Singlet$	$f_0(980)$		680	980±10	31	
$\tilde{u}\tilde{u}$	$a_0(980)$		680	984.8±1.4	31	S
$\tilde{s}\tilde{u}$	$K_0^*(960)$ (?)		1080	905±50 (?)	19 (?)	
$\tilde{s}\tilde{s}$	$f_0(1370)$		1360	1200-1500	(?)	
$(Singlet)'$	$f_0(1500)$		1350	1500±10	10	
$\tilde{u}'\tilde{u}'$	$a_0(1450)$		1350	1474±19	8	S'
$\tilde{s}'\tilde{u}'$	$K_0^*(1430)$		1440	1412±6	2	
$\tilde{s}'\tilde{s}'$	$f_0(1710)$		1530	1715±7	11	
$Singlet$	$\omega(782)$		770	782.57±0.12	2	
$\tilde{u}\tilde{u}$	$\rho(770)$	770		769.3±0.08		V
$\tilde{s}\tilde{u}$	$K^*(892)$		900	892-896	<1	
$\tilde{s}\tilde{s}$	$\varphi(1020)$	1020		1019.417±0.014		
$(Singlet)'$	$\omega(1420)$		1460	1419±31	3	
$\tilde{u}'\tilde{u}'$	$\rho(1450)$	1460		1465±25		V'
$\tilde{s}'\tilde{u}'$	$K^*(1410)$		1570	1414±15	11	
$\tilde{s}'\tilde{s}'$	$\varphi(1680)$	1680		1680±20		
$Singlet$	$f_1(1285)$		1120	1281.9±0.6	12	
$\tilde{u}\tilde{u}$	$a_1(1260)$		1120	1230±40	9	A
$\tilde{s}\tilde{u}$	$K_1(1400)$		1470	1402±7	5	
$\tilde{s}\tilde{s}$	$f_1(1510)$		1740	1512±4	15	
$(Singlet)'$	(?)		1520	(?)	(?)	
$\tilde{u}'\tilde{u}'$	$a_1(1640)$		1520	1640±40 (?)	7	A'
$\tilde{s}'\tilde{u}'$	$K_1(1650)$		1630	1650±50 (?)	1	
$\tilde{s}'\tilde{s}'$	(?)		1730	(?)	(?)	